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Testing the fermionic terms in the non-abelian D-brane effective action through order α'^3

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Abstract: Recently the construction of the non-abelian effective D-brane action was performed through order α'^3 including the terms quadratic in the gauginos. This result can be tested by calculating the spectrum in the presence of constant magnetic background fields and comparing it to the string theoretic predictions. This test was already performed for the purely bosonic terms. In this note we extend the test to the fermionic terms. We obtain perfect agreement.

Keywords: D-branes, Brane Dynamics in Gauge Theories.

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1. Introduction

While the abelian tree-level effective action for Dp-branes is known through all orders in α', at least in the limit of slowly varying fields, this is not so in the non-abelian case. In leading order the effective action for n coinciding Dp-branes is the ten-dimensional $N = 1$ supersymmetric U(n) Yang-Mills theory dimensionally reduced to $p+1$ dimensions. There are no $\mathcal{O}(\alpha')$ corrections. The bosonic $\mathcal{O}(\alpha'^2)$ were first obtained in [1] and [2] while the fermionic terms were obtained in [3] and [4]. In [3] supersymmetry fixed the correction while in [4] a direct calculation starting from four-point open superstring amplitudes was used. Requiring the existence of certain BPS configurations allowed for the determination of the bosonic $\mathcal{O}(\alpha'^3)$ terms in the effective action [5]. Just recently, in [6], supersymmetry was used not only to confirm the results of [5] but to construct the terms quadratic in the gauginos through this order as well.

Lacking direct string theoretic calculations, checks of these results are called for. In [7], further developed in [8] and [9], such a test was proposed. One starts from two D2p-branes wrapped around a p-dimensional torus. When switching on constant magnetic background fields this yields, upon T-dualizing, two intersecting Dp-branes. String theory allows for the calculation of the spectrum of strings stretching between different branes [10, 11]. In the context of the effective action, the spectrum should be reproduced by the mass spectrum of the off-diagonal field fluctuations. In [12] it was shown that the bosonic terms through $\mathcal{O}(\alpha'^3)$ correctly reproduce the spectrum of the gauge fields. In the present paper we will extend this analysis to the terms quadratic in the gauginos. Throughout the paper we will put $2\pi\alpha' = 1$ and we will follow the conventions of [6].

2. The spectrum from string theory

We consider a constant magnetic background on two coincident D2p-branes,

$$\mathcal{F}_{2a-1\,2a} = i \begin{pmatrix} f_a & 0 \\ 0 & -f_a \end{pmatrix} . \tag{2.1}$$

with $a \in \{1, 2, \ldots, p\}$ and $f_a \in \mathbb{R}$, $f_a > 0$. We choose a gauge such that $\mathcal{A}_{2a-1} = 0$, $\forall a$, and T-dualize in the 2, 4, ..., $2p$ directions. We end up with two intersecting Dp-branes. We want to calculate the spectrum of open strings stretching between the two branes. We take the first brane located along the 1, 3, ..., $2p-1$ directions. The other brane has been rotated with respect to the first one over an angle θ_1 in the 12 plane, over an angle θ_2 in the 34 plane, ..., over an angle θ_p in the $2p-1\,2p$ plane. The angles are determined by the magnetic fields,

$$\theta_a = 2 \arctan f_a \,, \qquad \forall a \in \{1, 2, \ldots, p\}. \tag{2.2}$$

Inspired by [11], we introduce,

$$\begin{aligned} \hat{X}^{2a-1} &= \cos\theta_a X^{2a-1} + \sin\theta_a X^{2a} \,, & \hat{X}^{2a} &= -\sin\theta_a X^{2a-1} + \cos\theta_a X^{2a} \,, \\ \hat{\psi}^{2a-1}_\pm &= \cos\theta_a \psi^{2a-1}_\pm + \sin\theta_a \psi^{2a}_\pm \,, & \hat{\psi}^{2a}_\pm &= -\sin\theta_a \psi^{2a-1}_\pm + \cos\theta_a \psi^{2a}_\pm \,, \end{aligned} \tag{2.3}$$

we impose the boundary conditions,

$$\begin{aligned} \text{at } \sigma = 0: \quad \partial_\sigma X^{2a-1} &= 0 \,, & \partial_\tau X^{2a} &= 0 \,, \\ \psi^{2a-1}_+ &= \psi^{2a-1}_- \,, & \psi^{2a}_+ &= -\psi^{2a}_- \,; \\ \text{at } \sigma = \pi: \quad \partial_\sigma \hat{X}^{2a-1} &= 0 \,, & \partial_\tau \hat{X}^{2a} &= 0 \,, \\ \hat{\psi}^{2a-1}_+ &= \eta \hat{\psi}^{2a-1}_- \,, & \hat{\psi}^{2a}_+ &= -\eta \hat{\psi}^{2a}_- \,, \end{aligned} \tag{2.4}$$

where $\eta = +1$ or $\eta = -1$ in the Ramond and the Neveu-Schwarz sector resp. Upon solving the equations of motion and implementing the boundary conditions we get the following expansion for the bosons,

$$\begin{aligned} X^{2a-1} &= \frac{i}{\sqrt{2\pi}} \sum_{n \in \mathbb{Z}} \left(\frac{\alpha_{n_{+a}}}{n_{+a}} e^{-in_{+a}\tau} \cos n_{+a}\sigma + \frac{\alpha_{n_{-a}}}{n_{-a}} e^{-in_{-a}\tau} \cos n_{-a}\sigma \right) , \\ X^{2a} &= \frac{i}{\sqrt{2\pi}} \sum_{n \in \mathbb{Z}} \left(\frac{\alpha_{n_{+a}}}{n_{+a}} e^{-in_{+a}\tau} \sin n_{+a}\sigma - \frac{\alpha_{n_{-a}}}{n_{-a}} e^{-in_{-a}\tau} \sin n_{-a}\sigma \right) , \end{aligned} \tag{2.5}$$

where we introduced

$$\varepsilon_a \equiv \frac{\theta_a}{\pi} \,, \qquad n_{\pm a} \equiv n \pm \varepsilon_a \text{ with } n \in \mathbb{Z} \,. \tag{2.6}$$

In the Ramond sector (we do not need the Neveu-Schwarz sector for this paper), we get

$$\begin{aligned} \psi^{2a-1}_\pm &= \frac{1}{2} \sum_{n \in \mathbb{Z}} \left(d_{n_{+a}} e^{-in_{+a}(\tau \pm \sigma)} + d_{n_{-a}} e^{-in_{-a}(\tau \pm \sigma)} \right) , \\ \psi^{2a}_\pm &= \pm \frac{i}{2} \sum_{n \in \mathbb{Z}} \left(d_{n_{+a}} e^{-in_{+a}(\tau \pm \sigma)} - d_{n_{-a}} e^{-in_{-a}(\tau \pm \sigma)} \right) . \end{aligned} \tag{2.7}$$

The non-vanishing (anti-)commutation relations are

$$\begin{aligned} [\alpha_{m_{+a}}, \alpha_{n_{-b}}] &= m_{+a}\delta_{m+n}\delta_{ab}\,, \\ \{d_{m_{+a}}, d_{n_{-b}}\} &= \delta_{m+n}\delta_{ab}\,. \end{aligned} \tag{2.8}$$

Both X^{2a-1} and X^{2a} contribute to the vacuum energy (in units where $2\pi\alpha' = 1$) by $-\pi/12 + \pi\varepsilon_a(1-\varepsilon_a)/2$ which is precisely cancelled by the contribution of the Ramond fermions. So just as for the case without magnetic fields, the vacuum energy vanishes in the Ramond sector. The (light-cone) states which in the absence of magnetic fields reduce to the gauginos are of the form

$$\prod_{a=1}^{p} (\alpha_{-\varepsilon_a})^{m_a} (d_{-\varepsilon_a})^{l_a} |0\rangle\,, \tag{2.9}$$

where $m_a \in \mathbb{N}$ and $l_a \in \{0,1\}$, $\forall a \in \{1,\ldots,p\}$ and $|0\rangle$ carries a chiral spinor representation of $Spin(8-2p)$. Their masses are given by

$$M^2 = \sum_{a=1}^{p} 2\,(m_a + l_a)\,\theta_a\,. \tag{2.10}$$

3. The spectrum from the effective action

3.1 The leading term

To set the stage we will first review some of the results of [13] and [14]. Our starting point is the U(2) $d=10$ $N=1$ supersymmetric Yang-Mills theory,[1]

$$\mathcal{L}_0 = -\frac{1}{g^2}\,\mathrm{Tr}\left\{-\frac{1}{4}F_{ab}F_{ab} + \frac{1}{2}\bar{\chi}\not{D}\chi\right\}. \tag{3.1}$$

For simplicity we will put $g=1$ throughout this paper. Compactifying $2p$ dimensions on a torus, we introduce complex coordinates for the compact directions, $z^\alpha = (x^{2\alpha-1} + ix^{2\alpha})/\sqrt{2}$, $\bar{z}^{\bar{\alpha}} = (z^\alpha)^*$, $\alpha \in \{1,\ldots,p\}$. We switch on constant magnetic background fields in the compact directions $\mathcal{F}_{\alpha\beta} = \mathcal{F}_{\bar{\alpha}\bar{\beta}} = 0$, $\mathcal{F}_{\alpha\bar{\beta}} = 0$ for $\alpha \neq \beta$ and[2]

$$\mathcal{F}_{\alpha\bar{\alpha}} = i\begin{pmatrix} f_\alpha & 0 \\ 0 & -f_\alpha \end{pmatrix}, \tag{3.2}$$

where the f_α, $\alpha \in \{1,\ldots,p\}$ are imaginary constants such that $if_\alpha > 0$. We only consider the off-diagonal components of the fermions,

$$\chi = i\begin{pmatrix} 0 & \chi^+ \\ \chi^- & 0 \end{pmatrix}, \tag{3.3}$$

[1] The calculation of the spectrum only probes U(2) sub-sectors of the full U(n) theory [14]. Note that we always write spacetime indices as lower indices.

[2] We do not sum over repeated indices corresponding to *complex* coordinates, unless indicated otherwise.

as the diagonal fluctuations probe the abelian part of the action. Using the previous choices, we can rewrite the second term in eq. (3.1) as,

$$\mathcal{L}_{\text{fermion}} = \bar{\chi}^{-}\left(\partial\!\!\!/_{NC} + \mathcal{D}\!\!\!\!/\right)\chi^{+}, \tag{3.4}$$

where subindex NC denotes operators acting in the non-compact directions only and $\mathcal{D} \equiv \partial + 2i\mathcal{A}$, with $\mathcal{A}$ the background gauge fields. The covariant derivatives satisfy

$$[\mathcal{D}_\alpha, \mathcal{D}_{\bar{\beta}}] = 2i\delta_{\alpha\beta} f_\alpha \,. \tag{3.5}$$

The equations of motion readily follow from eq. (3.4),

$$\left(\partial\!\!\!/_{NC} + \mathcal{D}\!\!\!\!/\right)\chi^{+} = 0\,. \tag{3.6}$$

Squaring the kinetic operator in eq. (3.6) and using eq. (3.5), we get,

$$\left(\Box_{NC} + 2\sum_{\alpha=1}^{p}\{\mathcal{D}_\alpha\mathcal{D}_{\bar{\alpha}} - if_\alpha - if_\alpha\gamma_{\alpha\bar{\alpha}}\}\right)\chi^{+} = 0\,, \tag{3.7}$$

where $\gamma_{\alpha\bar{\alpha}} \equiv (\gamma_\alpha\gamma_{\bar{\alpha}} - \gamma_{\bar{\alpha}}\gamma_\alpha)/2$, $(\gamma_{\alpha\bar{\alpha}})^2 = 1$. Once a complete set of eigenfunctions is constructed for the second part in eq. (3.7), we can bring the relation above in the form $(\Box - M^2)\chi = 0$ and read off the mass M. Such eigenfunctions are obtained from a spinor $|0\rangle$ satisfying $\mathcal{D}_{\bar{\alpha}}|0\rangle = 0$, $\forall\alpha$, which has been explicitly constructed in [13] and [14]. We now introduce the complete set of functions $|\{(m_1,n_1),(m_2,n_2),\ldots(m_p,n_p)\}\rangle$, $m_1, m_2, \ldots, m_p \in \mathbb{N}$ and $n_1, n_2, \ldots n_p \in \{-1,+1\}$ by

$$\begin{aligned}|\{(m_1,n_1),\,(m_2,n_2),\,\ldots,(m_p,n_p)\}\rangle \equiv{}& \frac{1}{2}(1+n_1\gamma_{1\bar{1}})\frac{1}{2}(1+n_2\gamma_{2\bar{2}})\cdots\frac{1}{2}(1+n_p\gamma_{p\bar{p}})\times\\ &\times\mathcal{D}_1^{m_1}\mathcal{D}_2^{m_2}\cdots\mathcal{D}_p^{m_p}|0\rangle\,.\end{aligned} \tag{3.8}$$

Expanding the fermion,

$$\chi^{+}(y,z,\bar{z}) = \sum_{\{(m,n)\}}\chi^{+}_{\{(m,n)\}}(y)|\{(m,n)\}\rangle\,, \tag{3.9}$$

where $\{(m,n)\} \equiv \{(m_1,n_1),(m_2,n_2),\ldots(m_p,n_p)\}$ and y collectively denotes the non-compact coordinates. Using this, one gets from eq. (3.5) and eq. (3.7) that the mass of $\chi^{+}_{\{(m,n)\}}(y)$ is given by

$$M^2 = 2i\sum_{\alpha=1}^{p}(2m_\alpha + 1 + n_\alpha)\,f_\alpha\,. \tag{3.10}$$

Replacing f_α by $\text{arctanh}(f_\alpha)$ in eq. (3.10) yields the stringy result, eq. (2.10). As expected, we only get agreement for very small magnetic background fields. The higher order terms in the effective action should add to this such that the string result gets reproduced. In particular one notices from this that only even orders in α' contribute to the spectrum.

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3.2 The $\mathcal{O}(\alpha'^2)$ contribution to the spectrum

Modulo field redefinitions and up to terms quartic in the fermions, the effective action through $\mathcal{O}(\alpha'^2)$ is given by $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2$ where $\mathcal{L}_0$ was given in eq. (3.1) and $\mathcal{L}_2$ is given by [1, 2, 3],

$$\begin{aligned}\mathcal{L}_2 = \mathrm{STr}\Big(& x_1 F_{ab}F_{ab}F_{cd}F_{cd} + x_2 F_{ab}F_{bc}F_{cd}F_{da} + \\ & + x_3 F_{ab}F_{ac}\bar{\chi}\gamma_b\mathcal{D}_c\chi + x_4 F_{ab}\mathcal{D}_a F_{cd}\bar{\chi}\gamma_{bcd}\chi\Big)\,, \end{aligned} \tag{3.11}$$

where STr denotes the symmetrized trace and

$$x_1 = -\frac{1}{32}\,, \qquad x_2 = \frac{1}{8}\,, \qquad x_3 = -\frac{1}{4}\,, \qquad x_4 = -\frac{1}{16}\,. \tag{3.12}$$

Again we want to calculate the fermionic spectrum through this order. It is clear that, as the background magnetic fields are (covariantly) constant, only the term proportional to x_3 will contribute. Following exactly the same strategy as above, we get the equations of motion,

$$\left(\not\partial_{NC} + \not{\mathcal{D}} - \frac{2x_3}{3}\sum_{\alpha=1}^{p} f_\alpha^2\left(\gamma_\alpha\mathcal{D}_{\bar{\alpha}} + \gamma_{\bar{\alpha}}\mathcal{D}_\alpha\right)\right)\chi^+ = 0\,. \tag{3.13}$$

Again squaring the kinetic operator we get,

$$\left(\Box_{NC} + 2\sum_{\alpha=1}^{p}\left(1 - \frac{4x_3}{3}f_\alpha^2\right)\left\{\mathcal{D}_\alpha\mathcal{D}_{\bar{\alpha}} - if_\alpha - if_\alpha\gamma_{\alpha\bar{\alpha}}\right\}\right)\chi^+ = 0\,, \tag{3.14}$$

where we ignored terms proportional to f^4 as they are of higher order in α'. However such terms will be relevant for a test of the, as of yet still unknown, $\mathcal{O}(\alpha'^4)$ terms in the effective action. It is clear that this gives the same spectrum as in eq. (3.10), but with f_α replaced by,

$$f_\alpha \to f_\alpha - \frac{4x_3}{3}f_\alpha^2\,. \tag{3.15}$$

Consistency with the string spectrum requires that $x_3 = -1/4$ which agrees with the result based on supersymmetry arguments and the direct calculation from open superstring amplitudes [3, 4].

In [9] it was shown that demanding that the spectrum of the gauge fields is correctly reproduced, combined with the requirement that the abelian limit agrees with the known result, completely fixes the bosonic part of the effective action through order α'^2. It is clear from the above that this is not the case for the fermionic terms which already indicates that the spectral test is indeed weaker for the terms containing fermions than for the purely bosonic terms.

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3.3 Testing the $\mathcal{O}(\alpha'^3)$ terms

At order $\mathcal{O}(\alpha'^3)$ the effective action is given by $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_3$, where $\mathcal{L}_0$ and $\mathcal{L}_2$ are given in eq. (3.1) and eq. (3.11). The last term is given by [6],[3] [5],

$$
\begin{aligned}
\mathcal{L}_3 = &-\frac{\zeta(3)}{16\pi^3} f^{XYZ} f^{VWZ} \Big[2\, F_{ab}{}^X F_{cd}{}^W \mathcal{D}_e F_{bc}{}^V \mathcal{D}_e F_{ad}{}^Y - 2 F_{ab}{}^X F_{ac}{}^W \mathcal{D}_d F_{be}{}^V \mathcal{D}_d F_{ce}{}^Y - \\
&\qquad + F_{ab}{}^X F_{cd}{}^W \mathcal{D}_e F_{ab}{}^V \mathcal{D}_e F_{cd}{}^Y - 4 F_{ab}{}^W \mathcal{D}_c F_{bd}{}^Y \bar\chi^X \gamma_a\, \mathcal{D}_d \mathcal{D}_c \chi^V - \\
&\qquad - 4 F_{ab}{}^W \mathcal{D}_c F_{bd}{}^Y \bar\chi^X \gamma_d\, \mathcal{D}_a \mathcal{D}_c \chi^V + 2 F_{ab}{}^W \mathcal{D}_c F_{de}{}^Y \bar\chi^X \gamma_{ade}\, \mathcal{D}_b \mathcal{D}_c \chi^V + \\
&\qquad + 2 F_{ab}{}^W \mathcal{D}_c F_{de}{}^Y \bar\chi^X \gamma_{abd}\, \mathcal{D}_e \mathcal{D}_c \chi^V \Big] - \\
&- \frac{\zeta(3)}{16\pi^3} f^{XYZ} f^{UVW} f^{TUX} \times \\
&\times \Big[4\, F_{ab}{}^Y F_{cd}{}^Z F_{ac}{}^V F_{be}{}^W F_{de}{}^T + 2\, F_{ab}{}^Y F_{cd}{}^Z F_{ab}{}^V F_{ce}{}^W F_{de}{}^T - \\
&\quad - 11\, F_{ab}{}^Y F_{cd}{}^Z F_{cd}{}^V \bar\chi^T \gamma_a\, \mathcal{D}_b \chi^W + 22\, F_{ab}{}^Y F_{cd}{}^Z F_{ac}{}^V \bar\chi^T \gamma_b\, \mathcal{D}_d \chi^W + \\
&\quad + 18\, F_{ab}{}^Y F_{cd}{}^V F_{ac}{}^W \bar\chi^T \gamma_b\, \mathcal{D}_d \chi^Z + 12\, F_{ab}{}^T F_{cd}{}^Y F_{ac}{}^V \bar\chi^Z \gamma_b\, \mathcal{D}_d \chi^W + \\
&\quad + 28\, F_{ab}{}^T F_{cd}{}^Y F_{ac}{}^V \bar\chi^W \gamma_b\, \mathcal{D}_d \chi^Z - 24\, F_{ab}{}^Y F_{cd}{}^V F_{ac}{}^T \bar\chi^W \gamma_b\, \mathcal{D}_d \chi^Z + \\
&\quad + 8\, F_{ab}{}^T F_{cd}{}^Y F_{ac}{}^Z \bar\chi^V \gamma_b\, \mathcal{D}_d \chi^W - 12\, F_{ab}{}^T F_{ac}{}^Y \mathcal{D}_b F_{cd}{}^V \bar\chi^Z \gamma_d\, \bar\chi^W - \\
&\quad - 8\, F_{ab}{}^Y F_{ac}{}^T \mathcal{D}_b F_{cd}{}^V \bar\chi^Z \gamma_d\, \bar\chi^W + 22\, F_{ab}{}^V F_{ac}{}^Y \mathcal{D}_b F_{cd}{}^T \bar\chi^Z \gamma_d\, \bar\chi^W - \\
&\quad - 4\, F_{ab}{}^Y F_{cd}{}^T \mathcal{D}_e F_{ac}{}^V \bar\chi^Z \gamma_{bde}\, \bar\chi^W + 4\, F_{ab}{}^Y F_{ac}{}^T \mathcal{D}_c F_{de}{}^V \bar\chi^Z \gamma_{bde}\, \bar\chi^W + \\
&\quad + 4\, F_{ab}{}^T F_{cd}{}^Y F_{ce}{}^V \bar\chi^Z \gamma_{abd}\, \mathcal{D}_e \chi^W - 8\, F_{ab}{}^Y F_{cd}{}^T F_{ce}{}^V \bar\chi^Z \gamma_{abd}\, \mathcal{D}_e \chi^W + \\
&\quad + 6\, F_{ab}{}^V F_{cd}{}^Y F_{ce}{}^W \bar\chi^Z \gamma_{abd}\, \mathcal{D}_e \chi^T + 5\, F_{ab}{}^V F_{cd}{}^W F_{ce}{}^Y \bar\chi^Z \gamma_{abd}\, \mathcal{D}_e \chi^T + \\
&\quad + 6\, F_{ab}{}^Y F_{ac}{}^T F_{de}{}^V \bar\chi^Z \gamma_{bcd}\, \mathcal{D}_e \chi^W - 2\, F_{ab}{}^Y F_{ac}{}^T F_{de}{}^Z \bar\chi^V \gamma_{bcd}\, \mathcal{D}_e \chi^W + \\
&\quad + 4\, F_{ab}{}^Y F_{ac}{}^V F_{de}{}^Z \bar\chi^W \gamma_{bcd}\, \mathcal{D}_e \chi^T + 4\, F_{ab}{}^T F_{cd}{}^V F_{ce}{}^Y \bar\chi^Z \gamma_{abd}\, \mathcal{D}_e \chi^W - \\
&\quad - 4\, F_{ab}{}^Y F_{cd}{}^V F_{ce}{}^W \bar\chi^Z \gamma_{abd}\, \mathcal{D}_e \chi^T + \frac{1}{2}\, F_{ab}{}^Y F_{cd}{}^T F_{ef}{}^V \bar\chi^Z \gamma_{abcde}\, \mathcal{D}_f \chi^W + \\
&\quad + \frac{1}{2}\, F_{ab}{}^Y F_{cd}{}^T F_{ef}{}^Z \bar\chi^V \gamma_{abcde}\, \mathcal{D}_f \chi^W \Big] .
\end{aligned}
\tag{3.16}
$$

The overall multiplicative constant can not be fixed by the methods used in either [5] or [6]. It gets determined through comparison with the relevant higher order derivative terms obtained in [15].

We now turn to the calculation of the spectrum. It is clear that terms involving a derivative on the field-strength will not contribute as the field-strength is covariantly constant. Furthermore, any term having two field-strengths contracted with a single f-symbol can be ignored as well as we took the background field-strength in the Cartan subalgebra of SU(2). Having discarded these terms we note that for the remaining terms the group theoretical factors, for our particular choice of background, are such that the Lie algebra indices on the gauginos are anti-symmetric when interchanging them. This implies that all terms involving a single or five gamma-matrices will vanish (up to a total derivative)

[3] We took U(n) generators in the fundamental representation satisfying $[t^X, t^Y] = f^{XYZ} t^Z$ where f^{XYZ} is completely anti-symmetric and $\mathrm{Tr}(t^X t^Y) = -\delta_{XY}$.

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as well. The only terms which can potentially contribute are now proportional to

$$\begin{aligned} & x_1\, F_{ab}{}^T F_{cd}{}^Y F_{ce}{}^V \bar{\chi}^Z \gamma_{abd}\, \mathcal{D}_e \chi^W + x_2\, F_{ab}{}^Y F_{cd}{}^T F_{ce}{}^V \bar{\chi}^Z \gamma_{abd}\, \mathcal{D}_e \chi^W + \\ & \qquad + x_3\, F_{ab}{}^Y F_{ac}{}^T F_{de}{}^V \bar{\chi}^Z \gamma_{bcd}\, \mathcal{D}_e \chi^W + x_4\, F_{ab}{}^T F_{cd}{}^V F_{ce}{}^Y \bar{\chi}^Z \gamma_{abd}\, \mathcal{D}_e \chi^W \,, \end{aligned} \tag{3.17}$$

where

$$x_1 = +4\,, \qquad x_2 = -8\,, \qquad x_3 = +6\,, \qquad x_4 = +4\,. \tag{3.18}$$

Rewriting eq. (3.17) in terms of the background and the off-diagonal fermions we get a result proportional to

$$(x_1 + x_2 + x_4) \sum_{\alpha=1}^{p} \sum_{\beta=1}^{p} f_\beta f_\alpha^2 \left(\bar{\chi}^- \gamma_{\bar{\beta}\beta\alpha} \mathcal{D}_{\bar{\alpha}} \chi^+ + \bar{\chi}^- \gamma_{\bar{\beta}\beta\bar{\alpha}} \mathcal{D}_\alpha \chi^+ \right), \tag{3.19}$$

which indeed vanishes when using eq. (3.18).

4. Conclusions

Though the spectral test is not as restrictive for the fermionic terms as it was for the purely bosonic terms, it is still gratifying to see that the fermionic terms pass it as well. The present proposal for the effective action through $\mathcal{O}(\alpha'^3)$ and up to terms quartic in the gauginos, is of the form,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{O}(\alpha'^4)\,, \tag{4.1}$$

where $\mathcal{L}_0$, $\mathcal{L}_2$ and $\mathcal{L}_3$ are given in eqs. (3.1), (3.11) and (3.16). The purely bosonic part of $\mathcal{L}_3$, which was obtained in [5], passed the spectral test in [12] while other proposals in the literature for these terms failed to do so. A very strong test of the purely bosonic terms was provided by the results in [6] where the supersymmetry invariant at order α'^3 was constructed. Its bosonic part precisely matches the one obtained in [5]. In addition, the terms quadratic in the gauginos were obtained as well. The bosonic and fermionic terms were tested in [6] by checking the closure of the commutator of two supersymmetry transformations. Furthermore, as was shown in the present paper, the fermionic terms correctly reproduce the gaugino spectrum in the presence of magnetic backgrounds.

So at this point there is no doubt left that we do have the correct description of the non-abelian D-brane effective action through order α'^3 and up to and including terms quadratic in the gauginos.

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